EPFL

Exercise III, Theory of Computation 2025

These exercises are for your own benefit. Feel free to collaborate and share your answers with other students. Solve as many problems as you can and ask for help if you get stuck for too long. Problems marked * are more difficult but also more fun :).

These problems are taken from various sources at EPFL and on the Internet, too numerous to cite individually.

1 Prove that the language $L = \{0^{n^2}1^n \mid n \geq 0\}$ is not regular.

2 Prove that the language $L = \{0^n 1^m \mid 0 \leq n \leq m \text{ OR } 0 \leq 2m \leq n\}$ is not regular.

3 Let L be language over the singleton alphabet $\{1\}$ consisting of all the strings whose lengths are prime numbers. Thus, $L = \{11, 111, 11111, 1111111, \ldots\}$. Prove that L is not regular.

4 A *palindrome* is a string that reads the same forwards and backwards.

Let L be the language over the alphabet $\{0, 1\}$ consisting of all the palindromes. For example, we have $00 \in L$, $10101 \in L$ and $\varepsilon \in L$. Prove that L is not regular.

5 For any string w over the alphabet $\{0, 1\}$, define it's *balance* by

$$\mathsf{bal}(w) = (\text{number of 1s in } w) - (\text{number of 0s in } w).$$

5a For some language L over the alphabet $\{0, 1\}$, define

$$B_L = \{w \in \{0,1\}^* \mid \mathsf{bal}(w) = \mathsf{bal}(u) \text{ for some } u \in L\}.$$

If L is regular, must B_L be regular as well? What about the other direction?

5b* Is the following language regular?

$$L = \{w \in \{0,1\}^* \mid -5 \leq \mathsf{bal}(s) \leq 5 \text{ for every substring } s \text{ of } w\}$$

Hint: *Keep track of the extremal values of* bal *for substrings ending at the current position.*

6* Let L be a language that satisfies the condition of the pumping lemma. Must L be regular?

Hint: *Modify the pumping lemma to look at the back of the string instead of the front.*